

# Transient Analysis of R-L & R-C Circuits

Complete Mathematical and Circuit Analysis under DC Excitations

# What is Transient Response?

## The Nature of Transients

- Definition: The temporary response of a circuit during the transition from one steady-state operating condition to another following a switching operation.
- Physical Cause: Sudden structural adjustments change the circuit energy configuration. Energy storage elements (Inductors and Capacitors) cannot alter their energy levels instantaneously.

## Mathematical Equation Context

**Total Response = Transient Response + Steady-State Response**

$$x(t) = x_t(t) + x_{ss}(t)$$

- Transient Part ( $x_t$ ): Governed by complementary functions of differential equations. Decays to zero as time goes to infinity.
- Steady-State Part ( $x_{ss}$ ): Governed by particular integrals. Represents the final stable state after switching effects vanish.

# Transient Response of Series R-L Circuit (DC Excitation)

## 1. Circuit Setup & KVL Equation

Consider a series combination of a Resistor (R) and Inductor (L) connected to a DC Voltage Source (V) via a switch closed at  $t = 0$ . Applying Kirchhoff's Voltage Law (KVL):

$$V = R \cdot i(t) + L \cdot [di(t) / dt]$$

## 2. Mathematical Derivation

- This is a 1st-order linear differential equation. Rearranging variables:  $di / (V - R \cdot i) = dt / L$
- Integrating both sides with the initial boundary condition  $i(0^+) = 0$  (inductor resists sudden current alterations):
- $-(1/R) \cdot \ln(V - R \cdot i) = t/L + K$ . Solving for current yields the standard charging equation:

$$i(t) = (V / R) \cdot [1 - e^{(-t / \tau)}] \text{ where Time Constant } \tau = L / R$$

# Transient Response of Series R-C Circuit (DC Excitation)

## 1. Circuit Setup & KVL Equation

Consider a series combination of a Resistor (R) and Capacitor (C) connected to a DC Voltage Source (V) via a switch closed at  $t = 0$ . Applying Kirchhoff's Voltage Law (KVL):

$$V = R \cdot i(t) + v_c(t) \Rightarrow V = R \cdot C \cdot [dv_c(t) / dt] + v_c(t)$$

## 2. Mathematical Derivation

- Separating variables for capacitor voltage:  $dv_c / (V - v_c) = dt / (R \cdot C)$
- Integrating both sides with initial uncharged boundary condition  $v_c(0^+) = 0$ :
- Solving for voltage yields the charging trajectory, and taking current as  $i(t) = C \cdot [dv_c / dt]$  yields current decay:

$$v_c(t) = V \cdot [1 - e^{(-t / \tau)}] \quad \text{and} \quad i(t) = (V / R) \cdot e^{(-t / \tau)} \quad \text{where Time Constant } \tau = R \cdot C$$

# Understanding the Time Constant ( $\tau$ )

| Time Interval | Growth Parameter [ $1 - e^{(-t/\tau)}$ ] (i in RL, $v_c$ in RC) | Decay Parameter [ $e^{(-t/\tau)}$ ] ( $v_L$ in RL, i in RC) |
|---------------|-----------------------------------------------------------------|-------------------------------------------------------------|
| $t = 1\tau$   | 63.2% of maximum value                                          | 36.8% of initial maximum value                              |
| $t = 2\tau$   | 86.5% of maximum value                                          | 13.5% of initial maximum value                              |
| $t = 3\tau$   | 95.0% of maximum value                                          | 5.0% of initial maximum value                               |
| $t = 5\tau$   | 99.3% of maximum value (Steady State)                           | 0.7% (Practically zero)                                     |

**Key Takeaway:** The transient response is theoretically infinite but practically considered over after 5 time constants ( $5\tau$ ), where the circuit reaches

# Solved Example 1: Series R-L Circuit

## Problem Statement:

A series circuit has  $R = 20 \, \Omega$  and  $L = 0.4 \, \text{H}$  connected to a constant DC source of  $V = 100 \, \text{V}$  via a switch closed at  $t = 0$ . Find:

(a) The time constant, (b) The expression for current  $i(t)$ , (c) Value of current at  $t = 0.04$  seconds.

## Step-by-Step Solution:

- Step (a): Find the Time Constant ( $\tau$ )

$$\tau = L / R = 0.4 \, \text{H} / 20 \, \Omega = 0.02 \, \text{seconds}$$

- Step (b): Determine the Current Equation  $i(t)$

$$\text{Maximum current steady-state value } I_{\text{max}} = V / R = 100 \, \text{V} / 20 \, \Omega = 5 \, \text{A}$$

$$\text{Using charging format: } i(t) = I_{\text{max}} \cdot [1 - e^{(-t/\tau)}] = 5 \cdot [1 - e^{(-t / 0.02)}] \, \text{A} \Rightarrow i(t) = 5 \cdot [1 - e^{(-50t)}] \, \text{A}$$

- Step (c): Evaluate Current at  $t = 0.04$  seconds

$t = 0.04 \, \text{s}$  represents exactly 2 time constants ( $2\tau$ ).

$$i(0.04) = 5 \cdot [1 - e^{(-50 \cdot 0.04)}] = 5 \cdot [1 - e^{(-2)}] = 5 \cdot [1 - 0.1353] = 5 \cdot 0.8647 = 4.3235 \, \text{A}$$

# Solved Example 2: Series R-C Circuit

## Problem Statement:

A series combination of  $R = 10 \text{ k}\Omega$  and  $C = 50 \text{ }\mu\text{F}$  is connected to a  $12 \text{ V}$  DC source at  $t = 0$ . Find:

(a) The time constant, (b) The capacitor voltage equation  $v_c(t)$ , (c) The initial circuit current at  $t = 0^+$ , (d) Time taken for voltage to reach  $9 \text{ V}$ .

## Step-by-Step Solution:

- Step (a): Find the Time Constant ( $\tau$ )

Convert components to base units:  $R = 10,000 \text{ }\Omega$ ,  $C = 50 \times 10^{-6} \text{ F}$

$$\tau = R \cdot C = 10,000 \cdot (50 \times 10^{-6}) = 0.5 \text{ seconds}$$

- Step (b): Determine Capacitor Voltage  $v_c(t)$

$$v_c(t) = V \cdot [1 - e^{(-t/\tau)}] = 12 \cdot [1 - e^{(-t / 0.5)}] \Rightarrow v_c(t) = 12 \cdot [1 - e^{(-2t)}] \text{ V}$$

- Step (c): Initial Current at  $t = 0^+$

At  $t = 0^+$ , the uncharged capacitor behaves like a short circuit.

$$i(0^+) = V / R = 12 \text{ V} / 10,000 \text{ }\Omega = 1.2 \text{ mA}$$

- Step (d): Calculate Time for  $v_c(t) = 9 \text{ V}$

$$9 = 12 \cdot [1 - e^{(-2t)}] \Rightarrow 0.75 = 1 - e^{(-2t)} \Rightarrow e^{(-2t)} = 0.25$$

$$-2t = \ln(0.25) = -1.3863 \Rightarrow t = 0.693 \text{ seconds}$$